Hierarchical Models

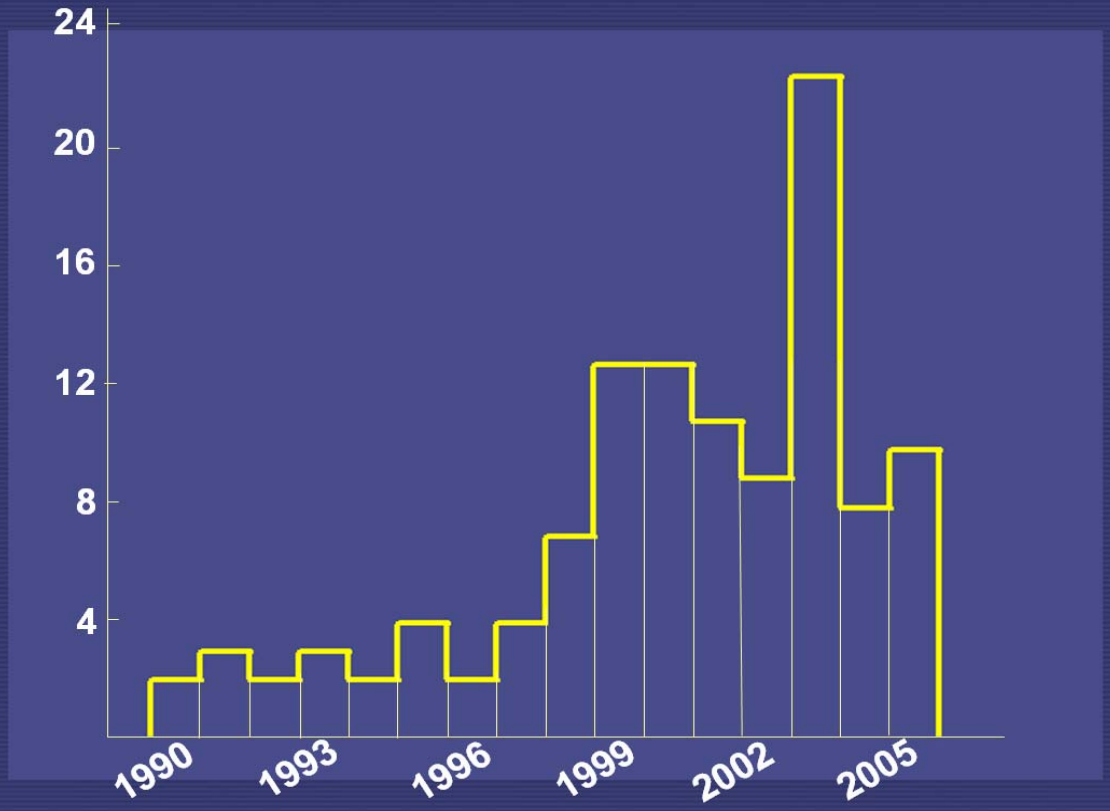
New frontiers in wildlife data analysis

USGS
science for a changing world

Hierarchical model publications

1990-2005: *Biometrics, Ecology, JWM, JABES*

(Keywords, title, abstracts)



Bayesian Analysis

- British mathematician and minister (1702-1761)
- “An introduction to the Doctrine of Fluxions, and a Defense of the Mathematicians against the objections of the author of *The Analyst* ” (1736)
- “Essay toward solving a Problem in the Doctrine of Chances” (1763, Richard Price)

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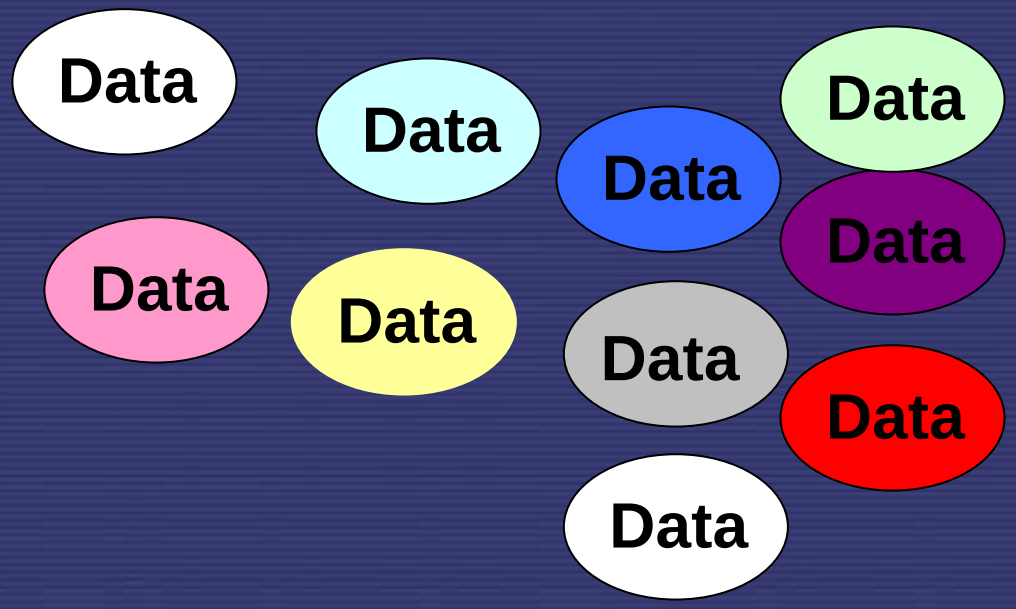
Why use models?

Data

Why use models?



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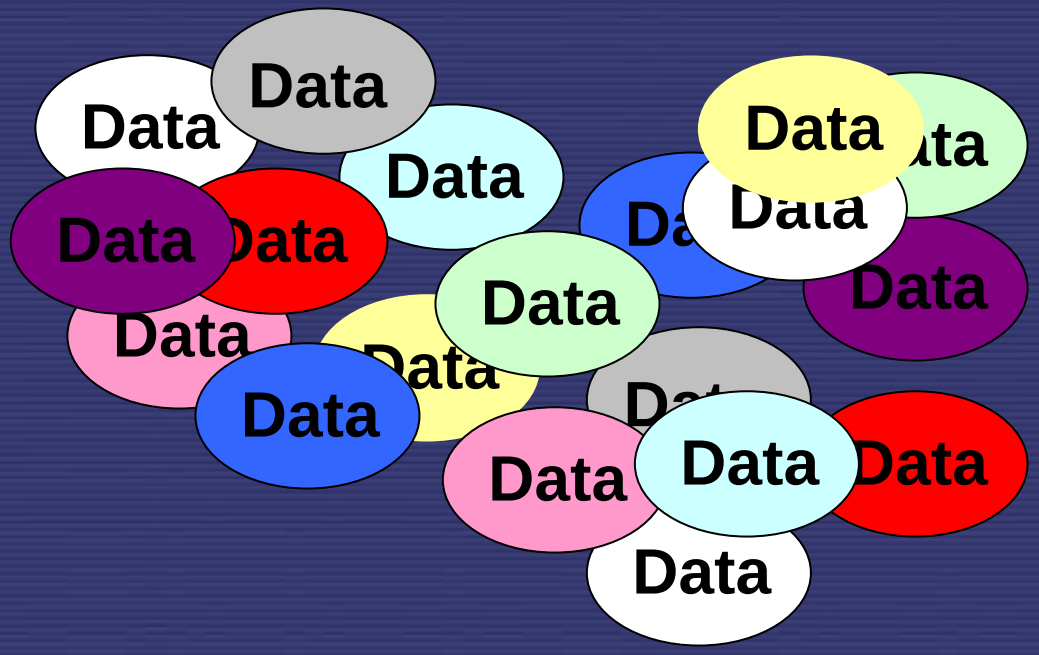

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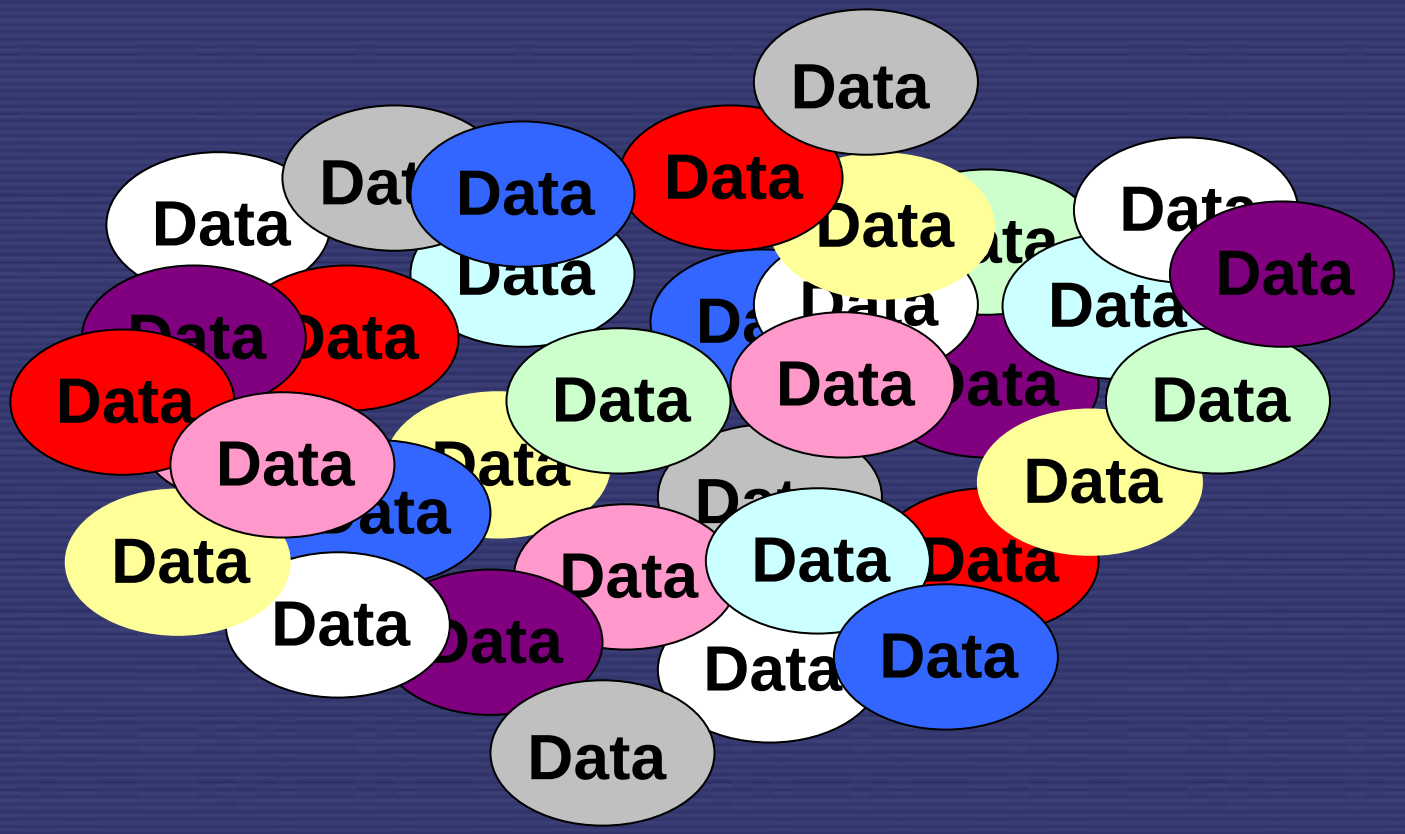

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Why use mode

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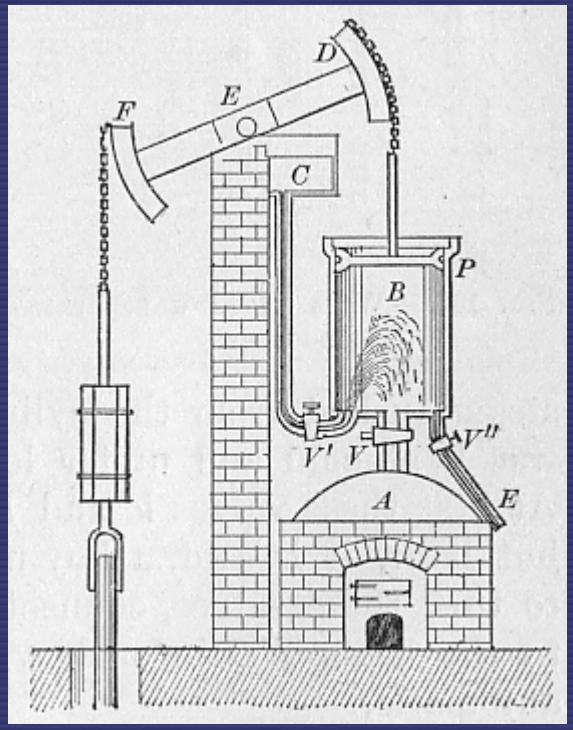
Probability

Parameters → 

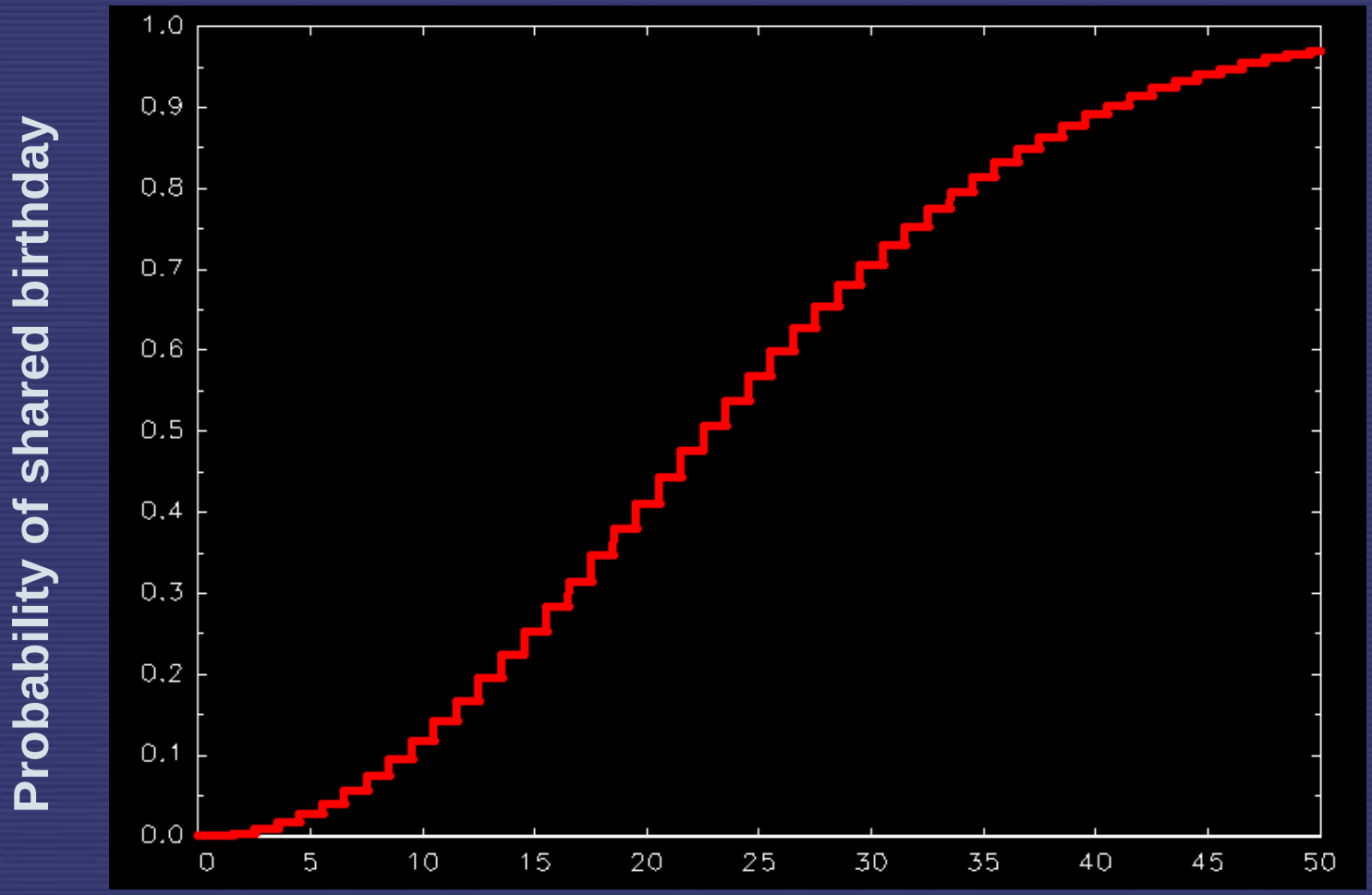
→ **Data**

Model

specified

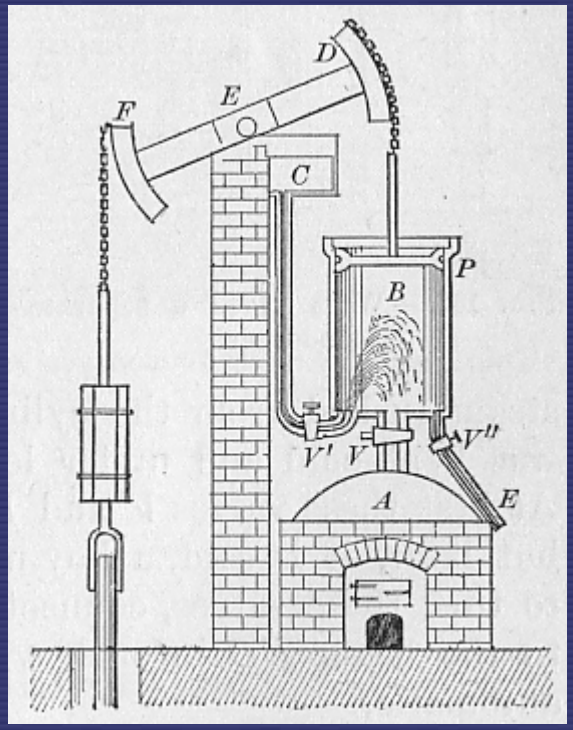
inferred

$$\Pr(\text{at least one match}) = 1 - \frac{N!}{(N-x)!\,N^x}$$



***x* = Number of people**

Probability

Parameters → 

→ **Data**

Model

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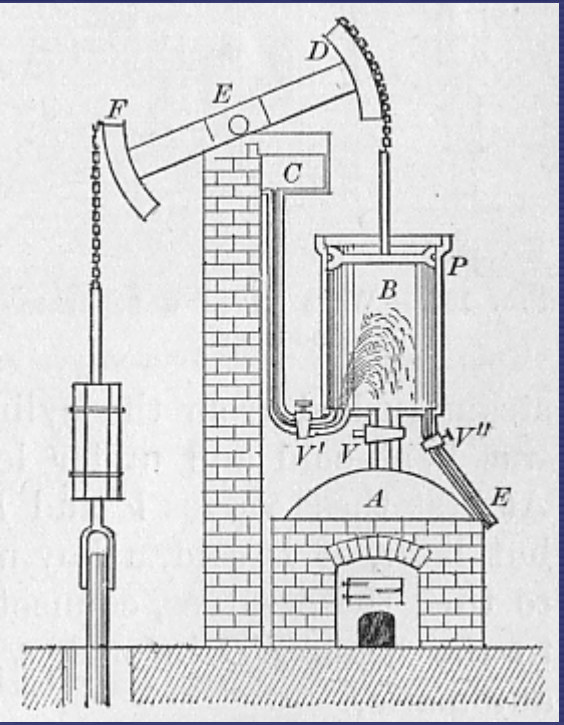
Statistics

Parameters



Data

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inferred

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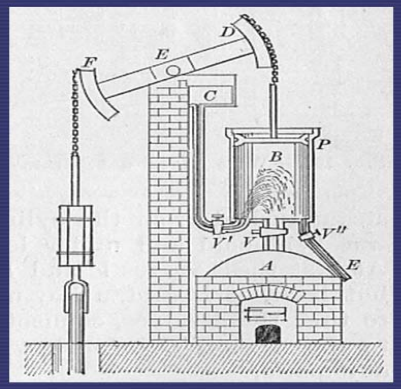
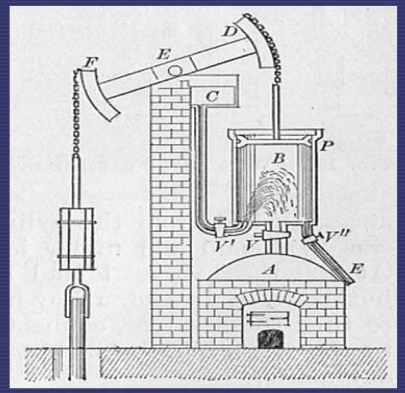
Probability

Parameters →  → Data

Model

Statistics

Parameters ←  ← Data

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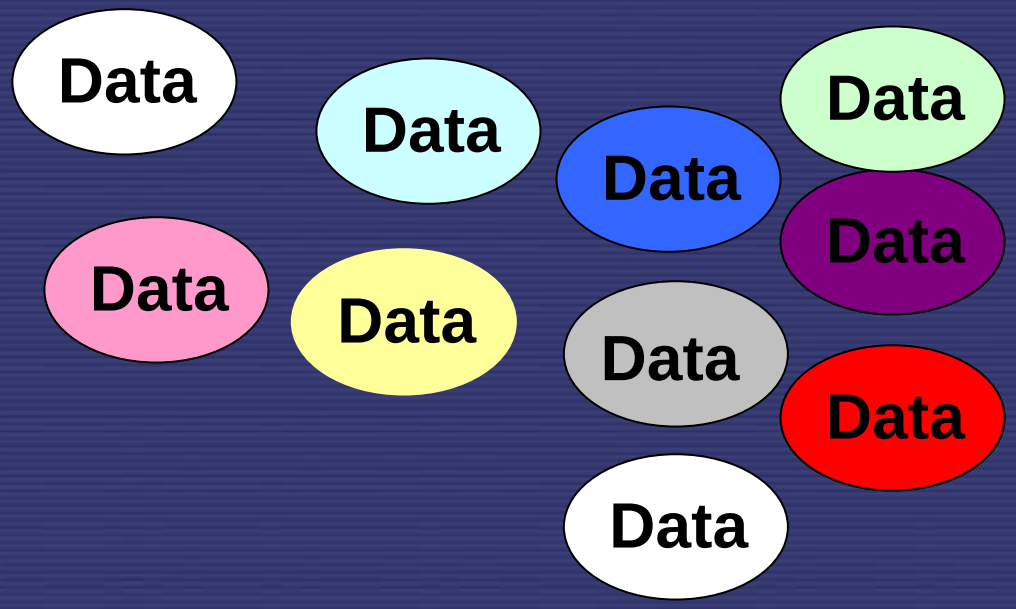
Why use models?

Data

Why use models?



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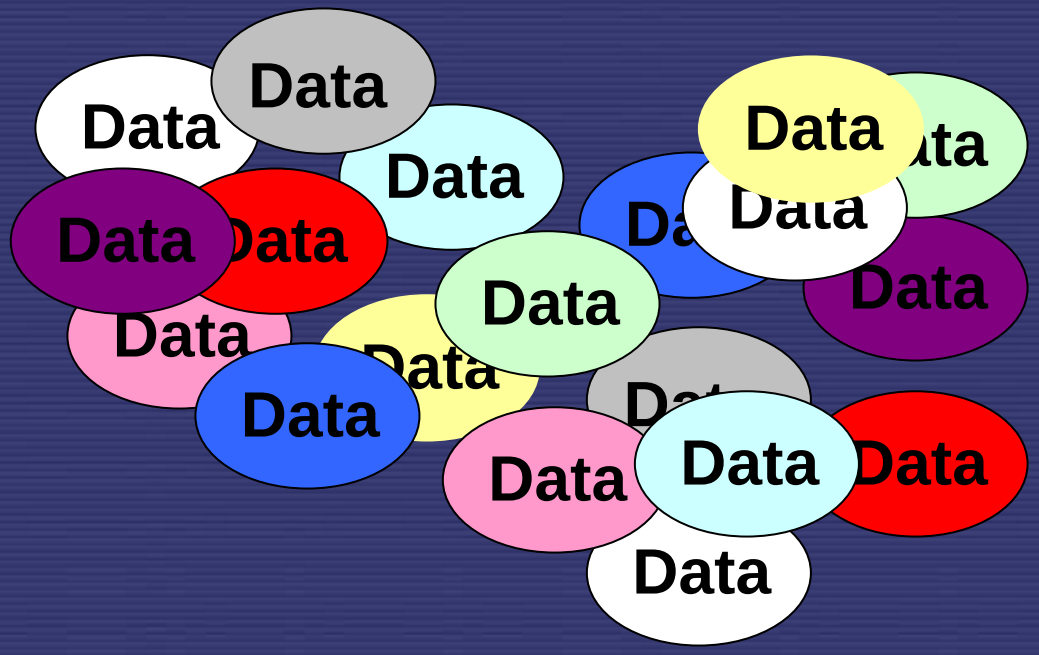

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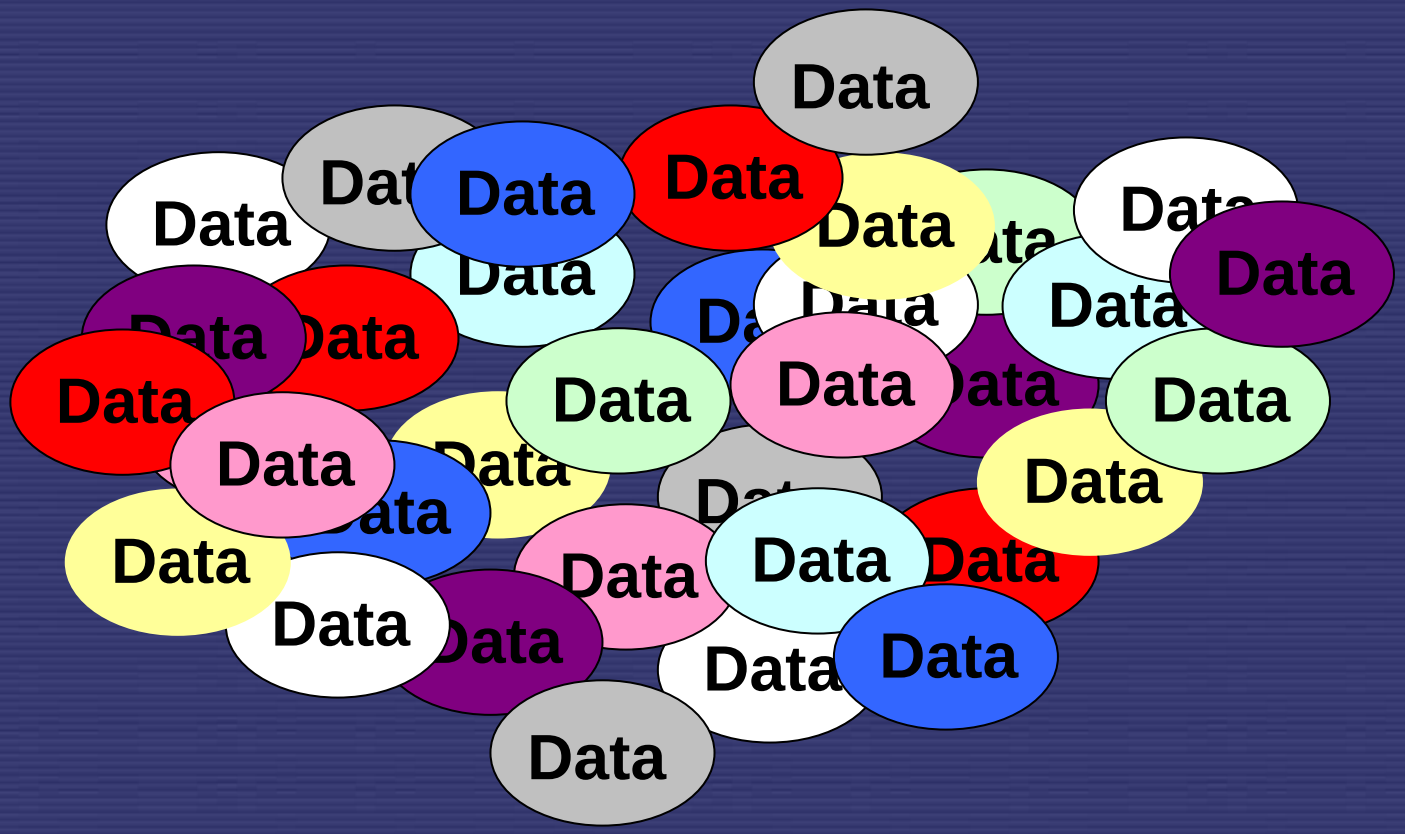

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Why use mode

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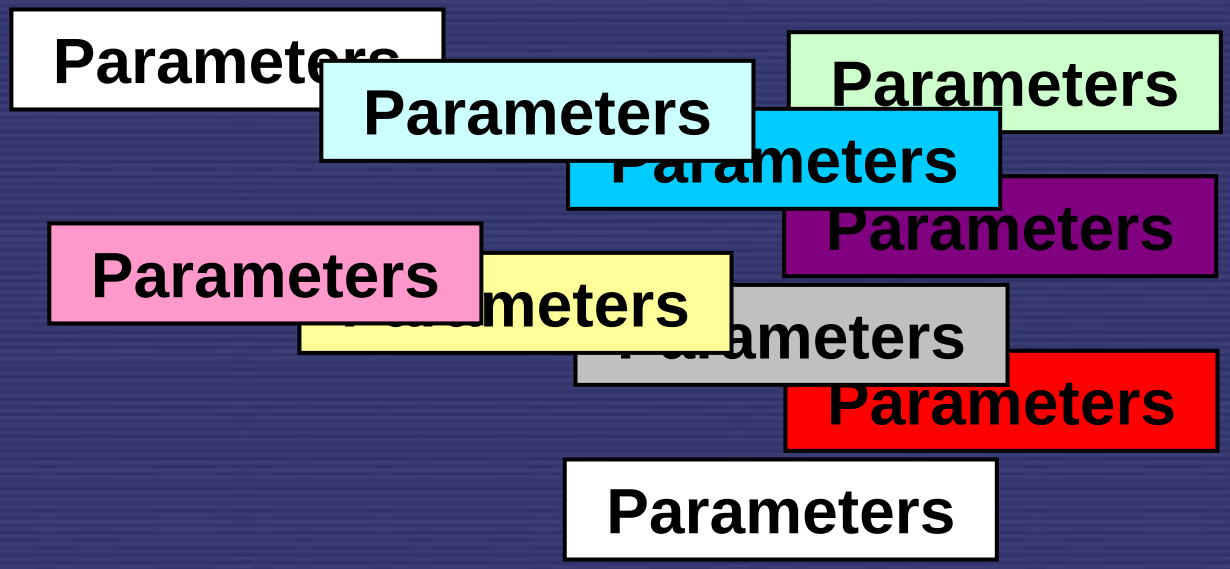
Why use hierarchical models?

Parameters

Why use hierarchical models?



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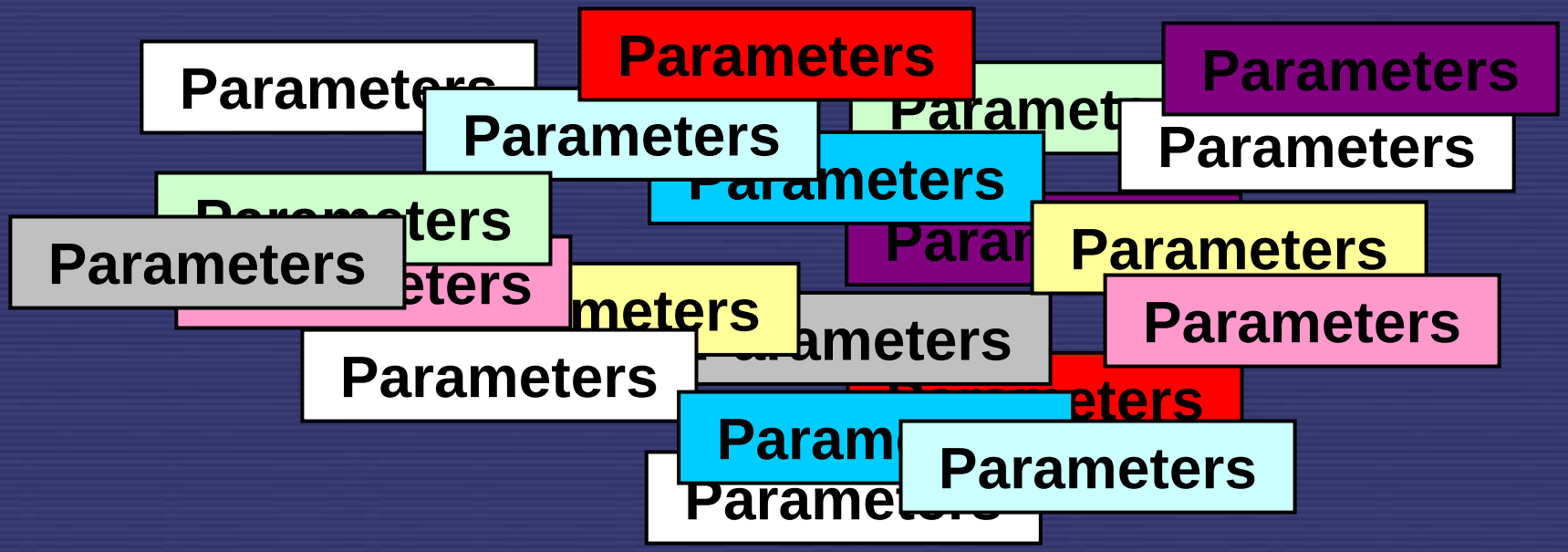

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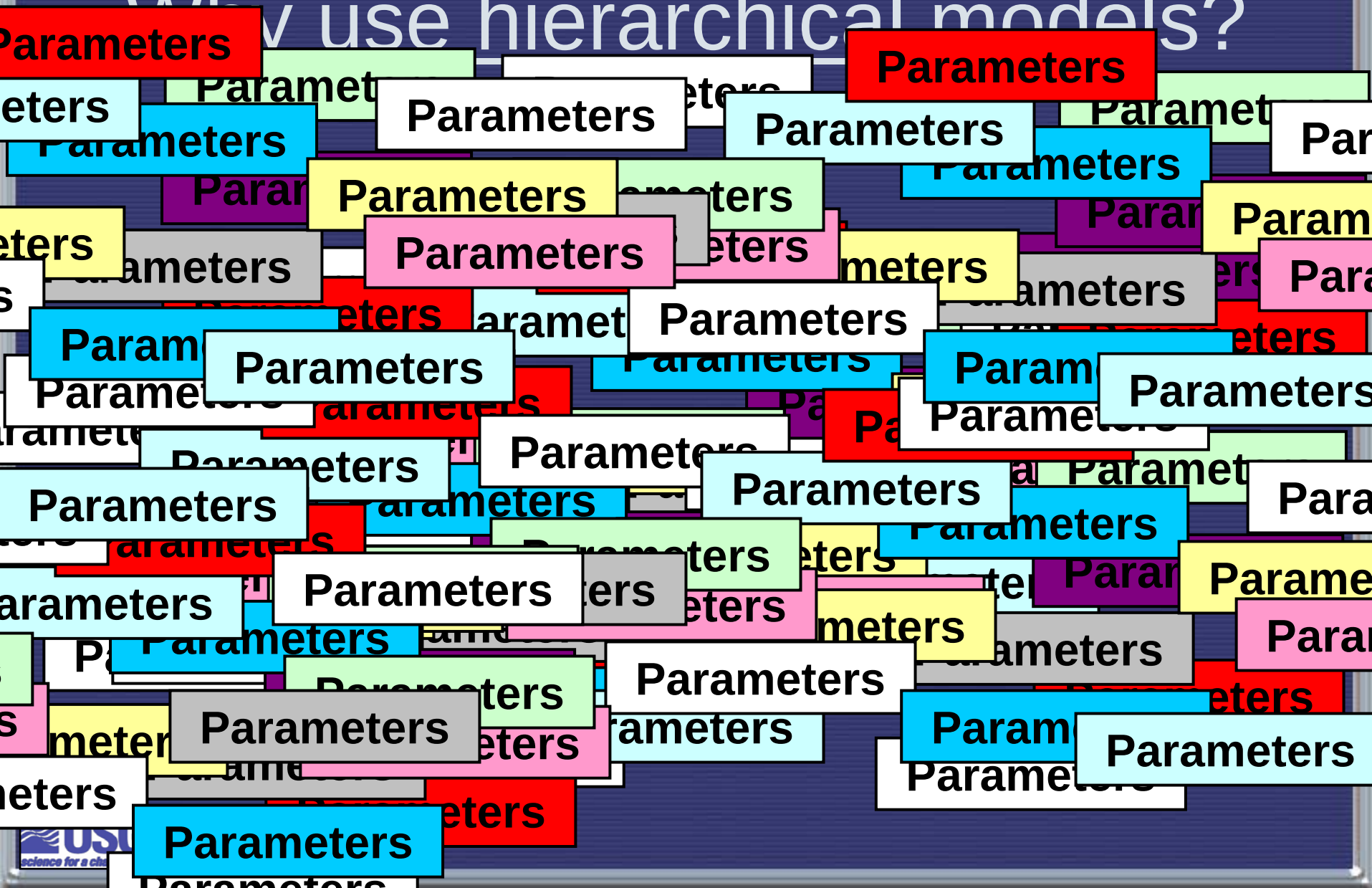
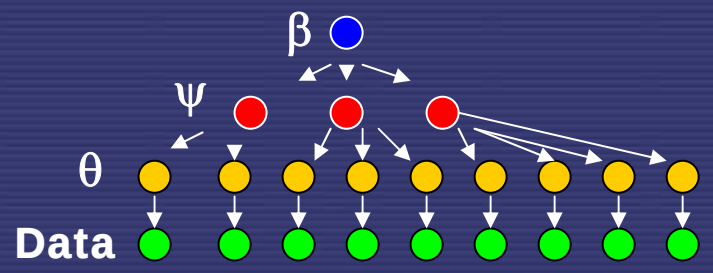
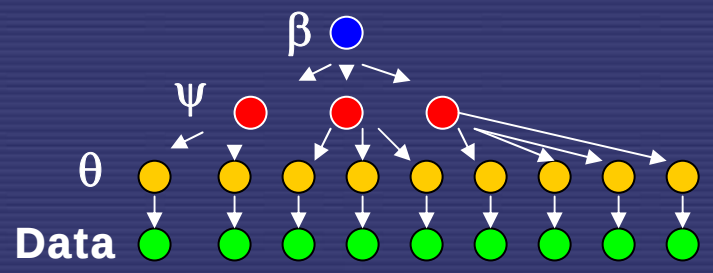


What is a hierarchical model?



- Observational model: *distribution of data* (also, missing data, future observables) *given parameters*
- Structural model: distribution of parameters, governed by *hyperparameters*
- Hyperparameter model

Hierarchical Models



- Investigate pattern among parameters
  - proper accounting of components of variation
  - “borrowing strength” allows improved estimates
- Examine latent structures
- Coherent framework for model selection

A simple example

- Bernoulli trials

A simple example

- Bernoulli trials

A simple example

- Bernoulli trials
- Nest success data

Observational Model

$$[Y_{sy} \mid p_{sy}] = \mathrm{B}(N_{sy}, p_{sy})$$

Structural Model

$$\mathrm{logit}(p_{sy}) = \mu + \mathrm{Site}_s + \mathrm{Year}_y + \varepsilon_{sy}$$

$$[\mathrm{Year}_y] = \mathrm{N}\left(\beta(y-4), \sigma_Y^2\right)$$

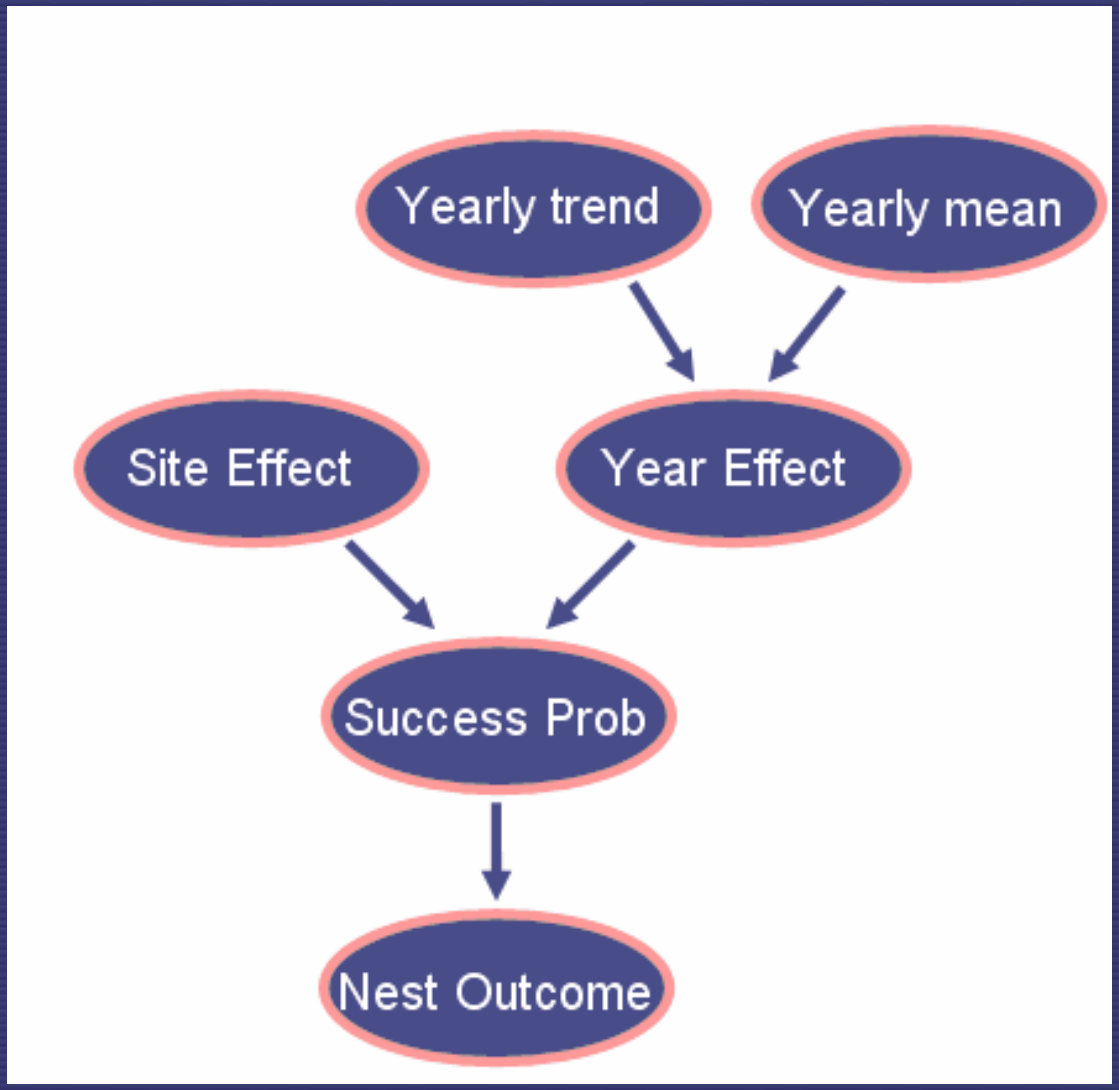
$$[\varepsilon_{sy}] = \mathrm{N}(0, \sigma_p^2)$$

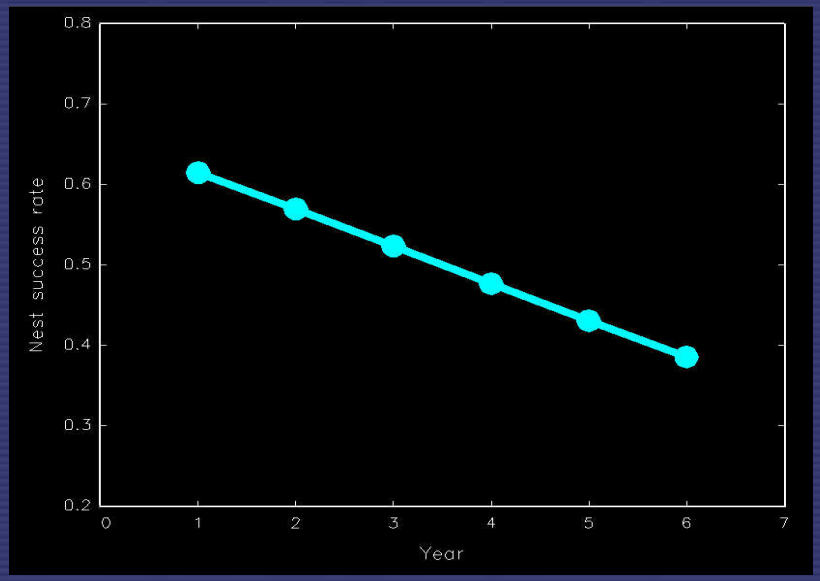
Hyperparameter Model

– *flat normal priors on* μ, Sites, β
– *flat inverse gamma priors on variances*

A simple example

- Bernoulli trials
- Nest success data

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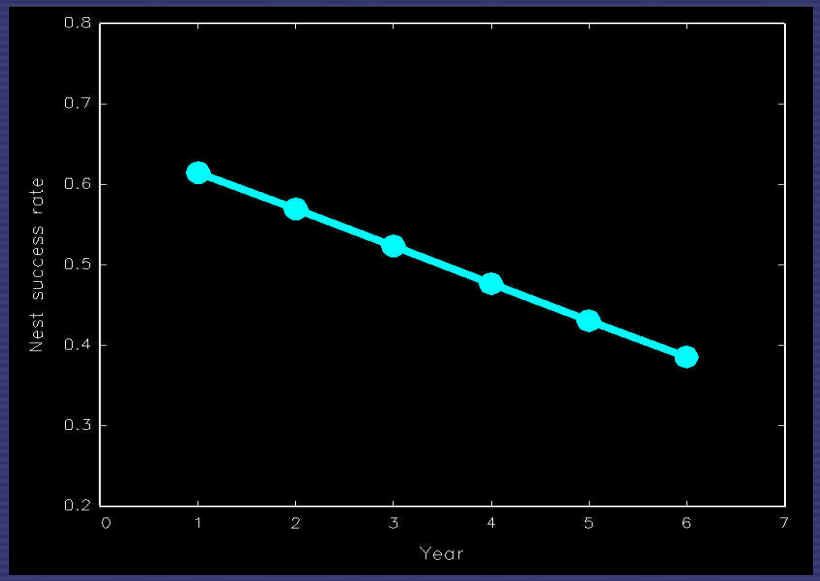
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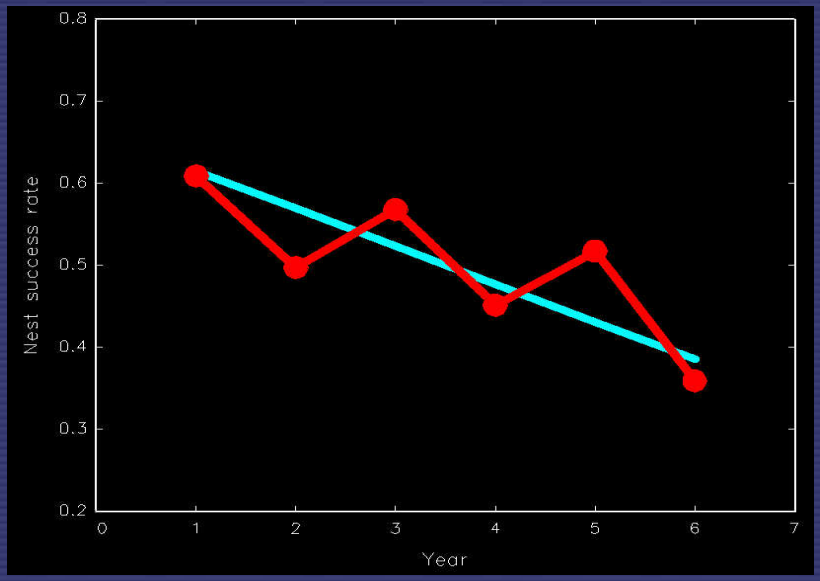
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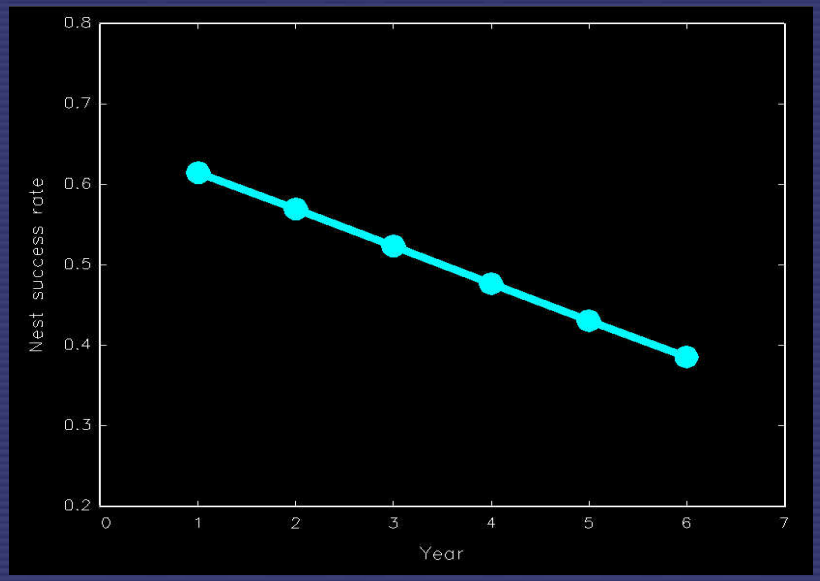
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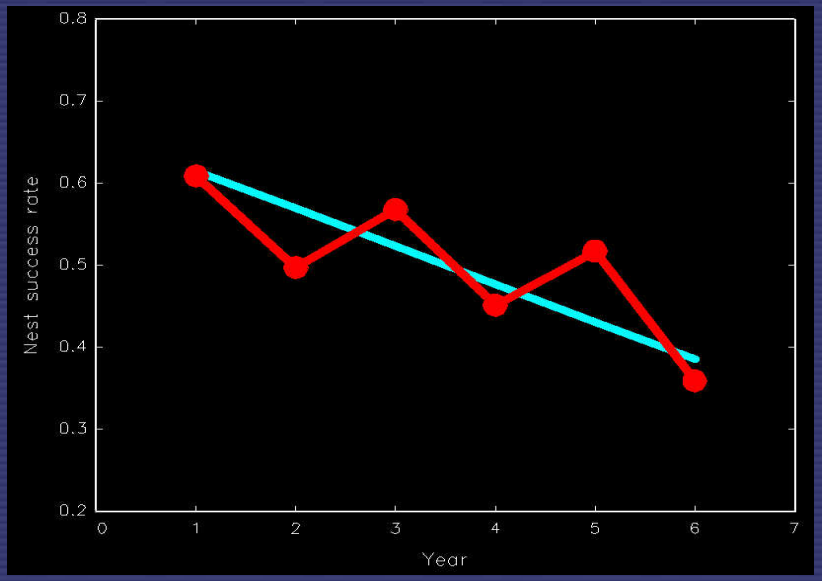
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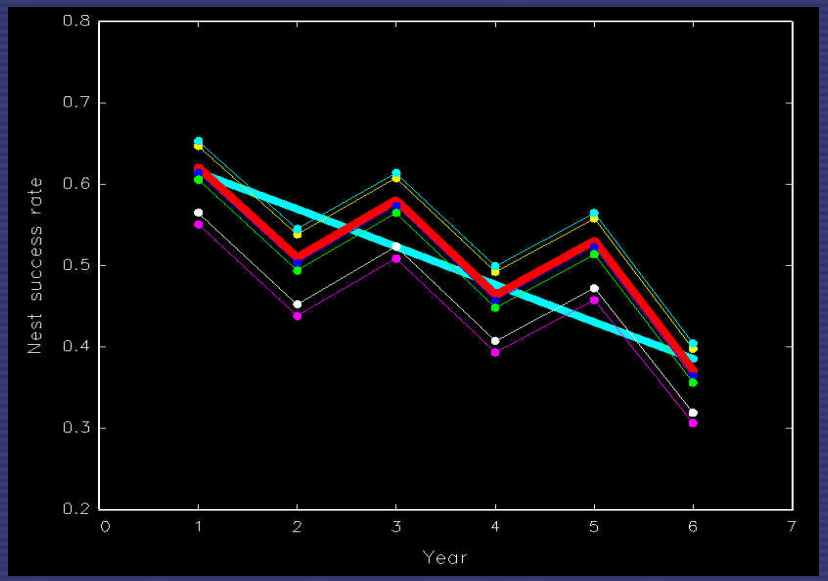
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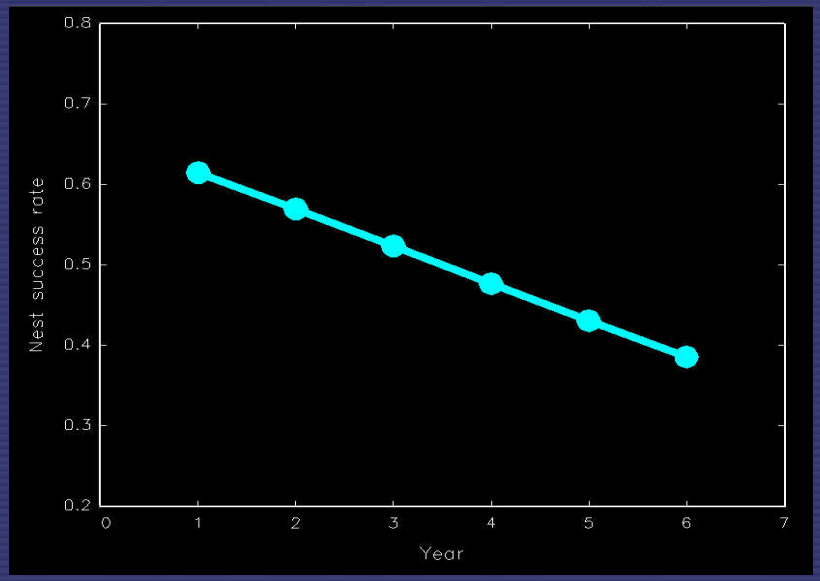
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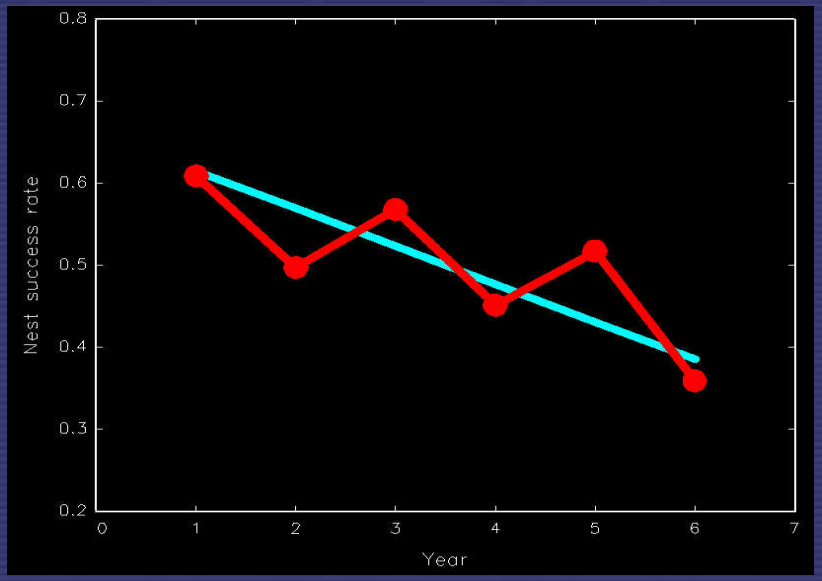
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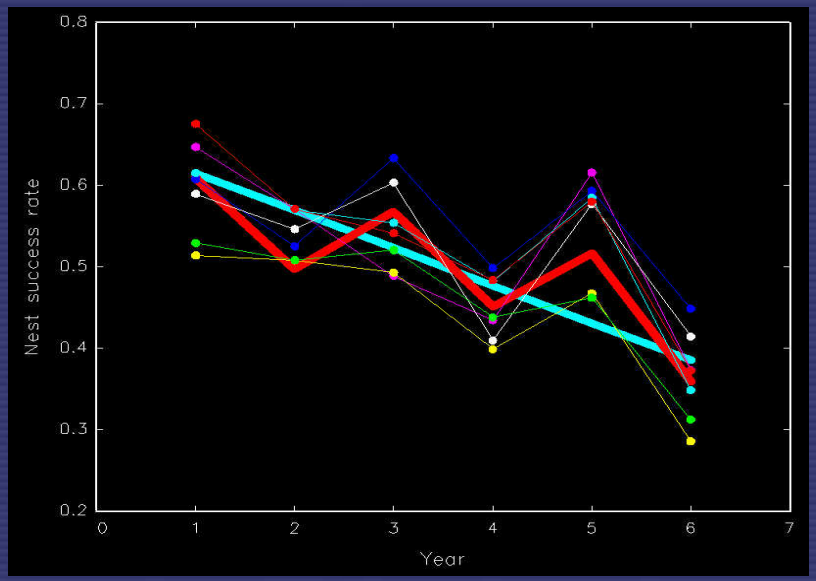
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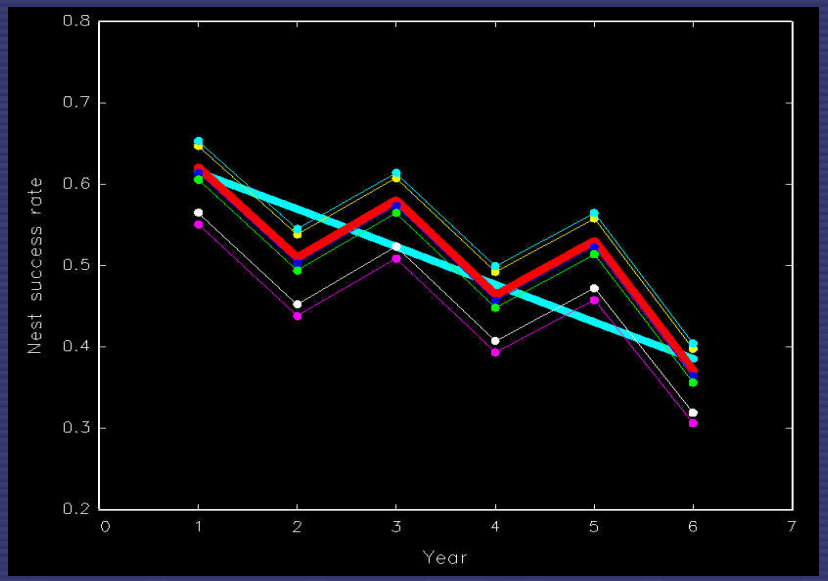
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BBS/CBC data

- Sauer, J.R. and Link, W.A. (2002) Hierarchical modeling of population stability and species group attributes from survey data. *Ecology* **86**(6):1743-1751.
- Link, W.A., and Sauer, J.R. (in press) Investigating seasonal components of avian population change through joint analysis of two large-scale monitoring programs. *Ecology*.
- Thogmartin W.E., J.R. Sauer, M.G. Knutson. 2004. A hierarchical spatial model of avian abundance with application to Cerulean warblers. *Ecological Applications* **14**: 1766–1779

Occupancy data

- Royle, J.A. and M. Kery. (in review) A Bayesian state-space formulation of dynamic occupancy models. *Ecology*.

Invasive species

- Wikle, C.K. 2003. Hierarchical Bayesian models for predicting the spread of ecological processes. *Ecology* **84**: 1382–1394.

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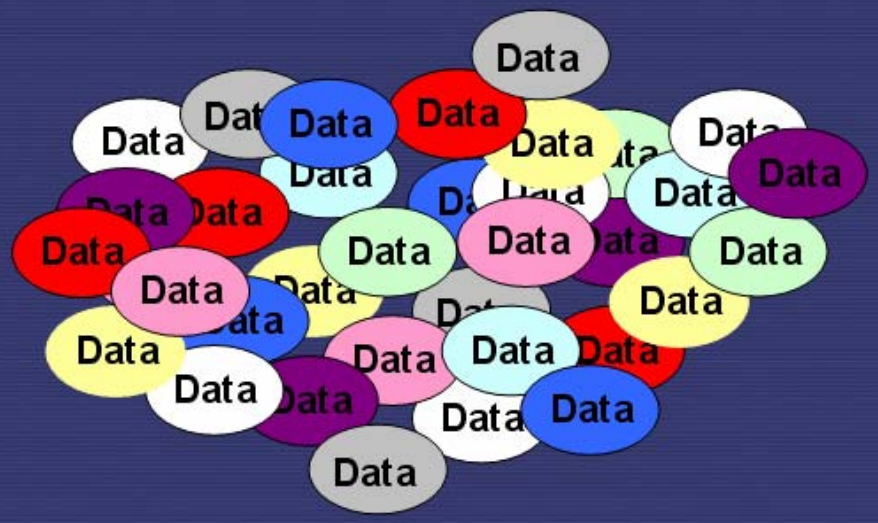

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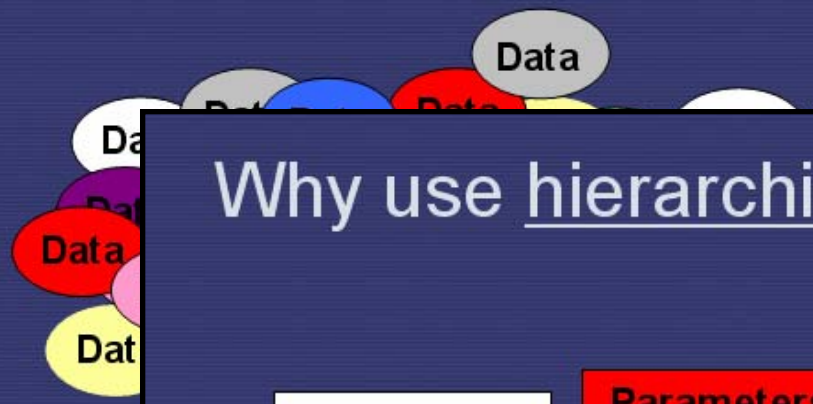

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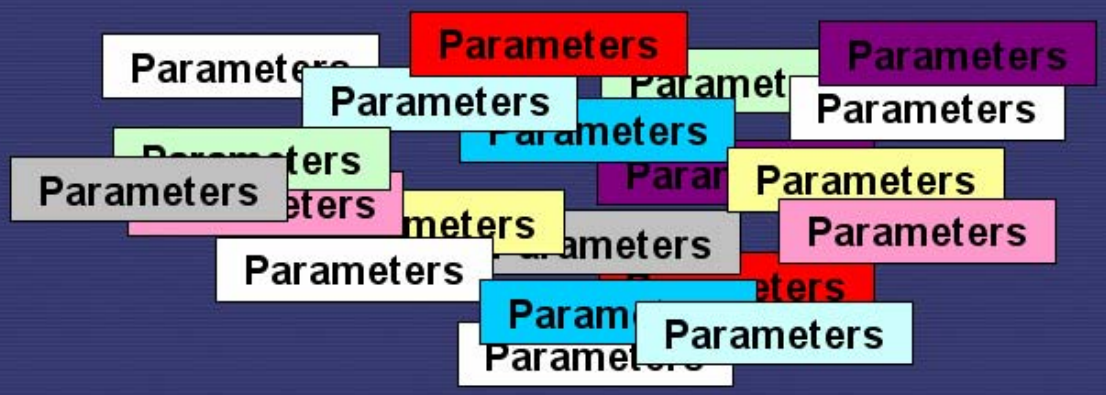

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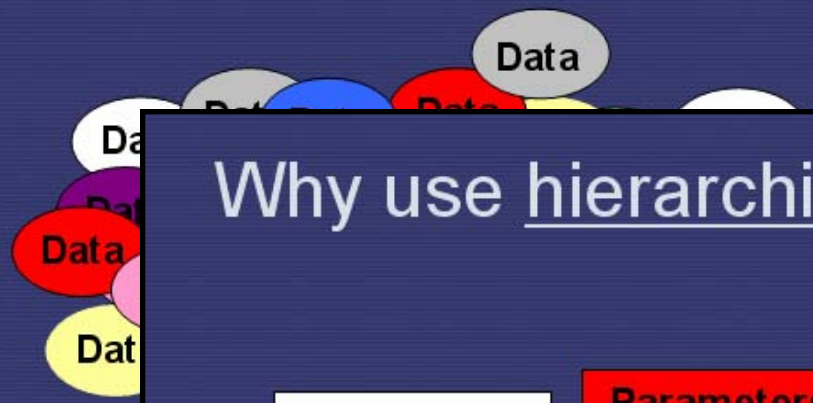
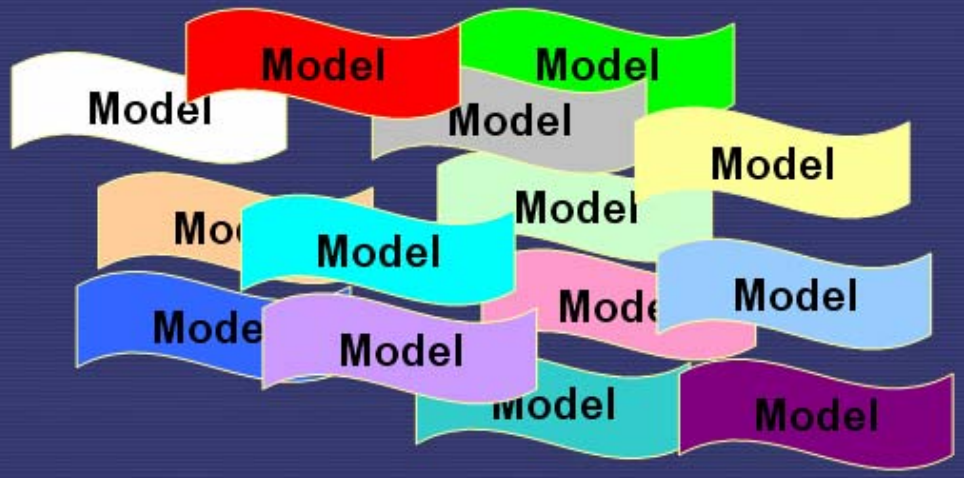


Why use hierarchical models?

What about model proliferation?



Model Uncertainty

- Which covariates?
- Which probability distribution?
- Which mathematical formulation?

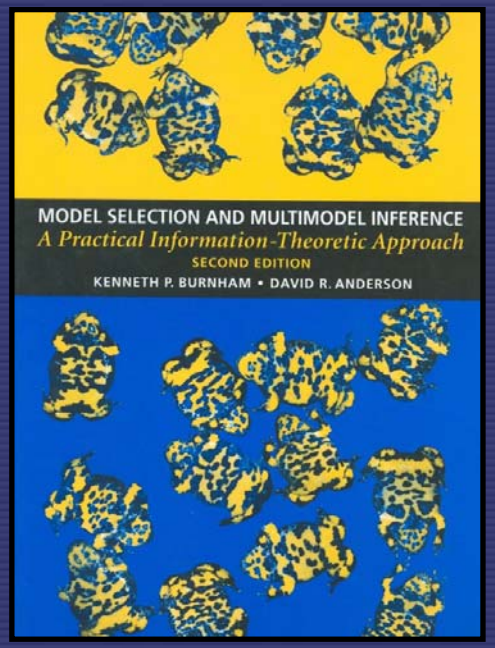
Model Uncertainty

- Which covariates?
- Which probability distribution?
- Which mathematical formulation?

$$Y \sim P(\lambda), \quad \log(\lambda) = \mathbf{X}\beta, \quad p = \Pr(Y > 0)$$

$$\Rightarrow \log\left(-\log(1-p)\right) = \mathbf{X}\beta$$

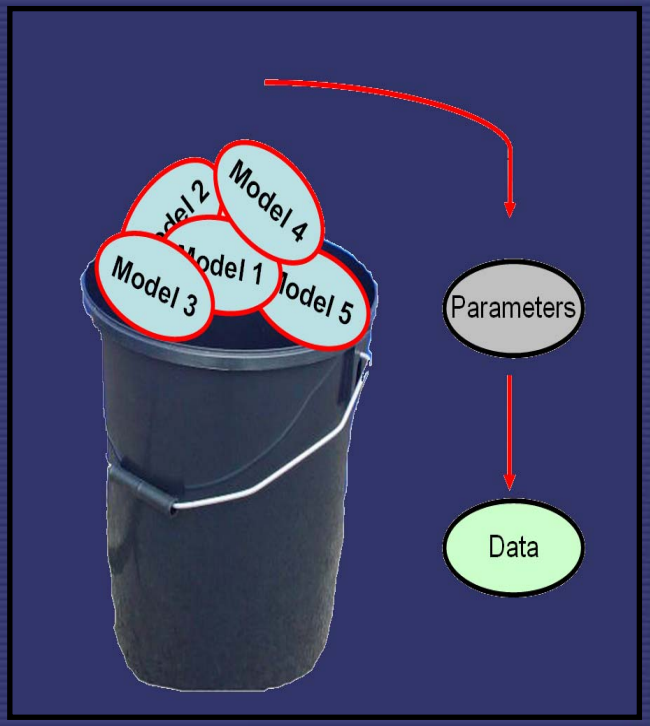
Model Uncertainty



AIC: Akaike's information criterion

- best virtue is simplicity
- some problems for random effects models
- asymptotic
- automatically favors highly parameterized models

Model Uncertainty



BMI: Bayesian multimodel inference

(Prior Odds) x (**Bayes Factor**) = Posterior Odds

Kestrel reproductive response to MeHg

Exposure	Number Eggs		
	0	1	2
1	0.1	0.4	0.5
2	0.1	0.4	0.5
3	0.1	0.4	0.5
4	0.6	0.3	0.1
5	0.6	0.3	0.1

or

	Number Eggs		
	0	1	2
1	0.1	0.4	0.5
2	0.3	0.3	0.4
3	0.4	0.3	0.3
4	0.5	0.3	0.2
5	0.7	0.2	0.1

?

USGS science for a changing world

Kestrel reproductive response to MeHg

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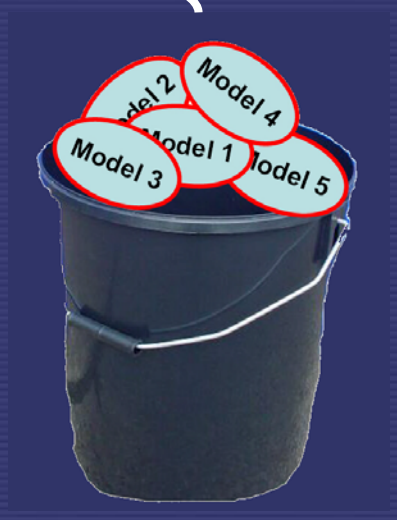
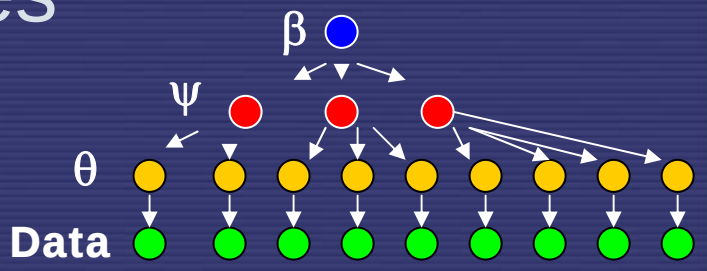
?

- BF = 2.44 against threshold model
- Equal prior model weights, posterior on threshold model = 29%
- 10:1 prior odds against threshold, posterior = 4%

USGS science for a changing world

Need a hierarchical model?



- Large data sets
- Missing values
- Latent structure, processes
- Model uncertainty